

CORIMIA IONIZATION MODEL – THREE INTERVAL APPROXIMATION FOR COSMIC RAY PROTONS ($Z = 1$)

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Keywords: Cosmic Ray Ionization, CORIMIA Model, Three-Interval Approximation, Energy Loss Function, Proton Ionization, Atmospheric Ionization

Abstract: The CORIMIA ionization model is used to describe the process of ionization in the Earth's atmosphere caused by cosmic ray particles. In this study, a three-interval approximation of the ionization loss function dE/dh is presented for cosmic ray protons with charge $Z = 1$. The approach extends the previous two-interval model by introducing a third interval for higher particle energies ($E_k > 5$ GeV), improving the accuracy of ionization rate calculations in the upper atmospheric regions. Analytical expressions are derived for the energy loss laws and the corresponding atmospheric cut-off parameters in each interval. The obtained formulation allows a more detailed estimation of the ionization yield and contributes to a better representation of the energy-dependent behavior of cosmic ray protons within the CORIMIA framework.

ЙОНИЗАЦИОНЕН МОДЕЛ CORIMIA – ТРИ-ИНТЕРВАЛНА АПРОКСИМАЦИЯ ЗА ПРОТОНИ ($Z = 1$)

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Ключови думи: Йонизация от Космически Лъчи, Модел CORIMIA, Три-Интервална Апроксимация, Функция На Енергийните Загуби, Йонизация от Протони, Атмосферна Йонизация

Резюме: Йонизационният Моделът CORIMIA се използва за описание на процеса на йонизация в земната атмосфера, предизвикан от космическите лъчи. В настоящото изследване е представена три-интервална апроксимация на функцията на йонизационните загуби dE/dh за космически лъчи протони със заряд $Z = 1$. Подходът разширява предишния дву-интервален модел чрез въвеждане на трети интервал за по-високи енергии на частиците ($E_k > 5$ GeV), като по този начин се повишава точността при изчисленията на скоростта на йонизация в горните атмосферни слоеве. Изведени са аналитични изрази за законите на енергийните загуби и съответните атмосферни прагове на отрязване за всеки интервал. Получената формулировка позволява по-прецизна оценка на йонизационния добив и по-добро представяне на енергийнозависимото поведение на протоните в състава на космическите лъчи.

Introduction

Cosmic rays are high-energy charged particles originating from galactic, solar, and interstellar sources. Upon entering the Earth's atmosphere, they interact with atmospheric constituents, producing cascades of secondary particles that contribute to atmospheric ionization and chemical changes [1]. The ionization caused by cosmic rays plays a fundamental role in atmospheric electricity, cloud microphysics, and long-term climate variability [2].

Depending on their origin and energy, cosmic rays are generally divided into three main components: Galactic Cosmic Rays (GCRs), Solar Energetic Particles (SEPs), and Anomalous Cosmic Rays (ACRs). The GCRs dominate during quiet solar conditions and provide the background ionization at all latitudes. SEPs are transient, event-related fluxes associated with solar flares and

coronal mass ejections, while ACRs represent a lower-energy component accelerated at the heliospheric boundary [3÷5].

The differential energy spectra of these components are shown in Figure 1, where the typical fluxes of GCRs during solar maximum and minimum are compared with representative SEP and ACR spectra. As illustrated, galactic protons and alpha particles contribute most significantly in the range between 0.1 and 10 GeV, which corresponds to the energy interval most relevant for atmospheric ionization processes.

Accurate modeling of cosmic ray ionization requires a detailed description of the energy loss of charged particles as they penetrate the atmosphere. The CORIMIA (Cosmic Ray Ionization Model for the Atmosphere) model provides such a framework by analytically integrating the energy loss function $(1/p)(dE/dh)$ over the primary energy spectrum, accounting for geomagnetic shielding and solar modulation effects [6]. The precision of this approach strongly depends on the adopted approximation of the ionization loss function, which determines the rate of energy dissipation with atmospheric depth.

Previous studies employed a two-interval approximation based on the Bethe–Bloch theory [7], successfully reproducing the observed ionization at low and medium energies. However, at higher kinetic energies ($E_k > 5$ GeV), deviations become noticeable. Therefore, a three-interval approximation has been developed to extend the applicability of CORIMIA for high-energy protons ($Z = 1$), providing better continuity and improved representation of the ionization loss function across a wide energy range [8].

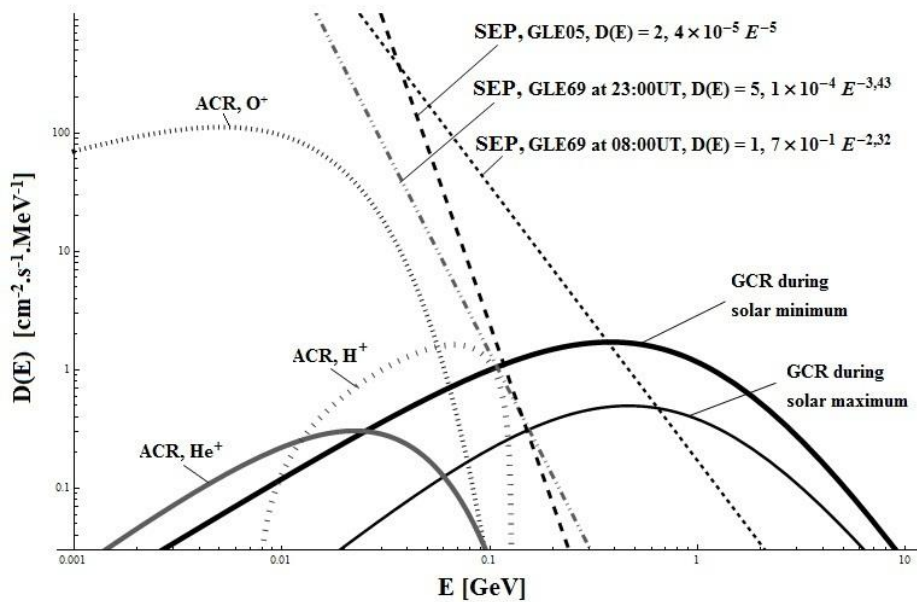


Fig. 1. Differential energy spectra of Galactic Cosmic Rays (GCR), Solar Energetic Particles (SEP), and Anomalous Cosmic Rays (ACR) for different particle species and solar conditions

Ionization Function and Model Formulation

The ionization rate in the Earth's atmosphere produced by cosmic ray particles is determined by their energy loss per unit mass thickness, expressed as:

$$(1) \quad q(h) = \left(\frac{\rho(h)}{Q} \right) \int_{E_{cut}}^{\infty} D(E) \left(\frac{dE}{dh} \right) dE,$$

where $\rho(h)$ is the atmospheric density at altitude h , $D(E)$ is the differential cosmic ray spectrum, Q is the mean energy required for the creation of an ion pair ($Q = 35$ eV), and E_{cut} is the effective cut-off energy determined by geomagnetic and atmospheric shielding.

The CORIMIA (COsmic Ray Ionization Model for the Atmosphere and Ionosphere) model evaluates $q(h)$ for various charged particles ($Z \geq 1$) using analytical expressions for the ionization loss function dE/dh . Following the approach of [3, 4], the model separates the particle energy range into characteristic intervals where the dependence of dE/dh on E can be approximated by power laws.

Energy Loss Function

For protons ($Z = 1$), the energy loss function is expressed as:

$$(2) \quad -\left(\frac{1}{\rho}\right)\left(\frac{dE}{dh}\right) = A_i \cdot E^{-\alpha_i}, \quad E_{i-1} \leq E < E_i,$$

where A_i and α_i are constants determined for each interval i . In the three-interval approximation, the limits are chosen as:

$$(3) \quad E_1 = 0.15 \text{ GeV}, \quad E_2 = 0.6 \text{ GeV}, \quad E_3 = 5.0 \text{ GeV}.$$

Thus, the function becomes:

$$(4) \quad -\left(\frac{1}{\rho}\right)\left(\frac{dE}{dh}\right) = \begin{cases} 242 \cdot E^{-\frac{3}{4}}, & 0.15 \leq E < 0.6 \text{ GeV} & \text{Interval 1} \\ 2, & 0.6 \leq E < 5.0 \text{ GeV} & \text{Interval 2} \\ 0.7 \cdot E^{0.123}, & E \geq 5.0 \text{ GeV} & \text{Interval 3} \end{cases}$$

These intervals correspond respectively to low-energy, intermediate, and high-energy domains of cosmic ray protons.

Penetration Depth and Atmospheric Cut-off

The quantity of matter traversed by a particle is given by the integral:

$$(5) \quad h = \int_{E_k}^{E_c} \left(\frac{1}{\rho}\right)\left(\frac{dE}{dh}\right) dE,$$

which defines the penetration depth as a function of the particle's initial energy E_k . Using the above approximation, analytical expressions for $h(E_k)$ are derived for each interval. For example, for $E_k \geq 5 \text{ GeV}$:

$$(6) \quad h = h_3 = \left(\frac{1}{0.7}\right) \int_{E(h)}^{E_k} \left(\frac{dE}{E^{0.123}}\right) = \left(\frac{1}{0.7 \times 0.877}\right) (E_k^{0.877} - E_3^{0.877}(h)),$$

and the inverse relation gives the energy of the particle at a given depth:

$$(7) \quad E_3(h) = (E_k^{0.877} - 0.61 \cdot h)^{1.14}.$$

These relations provide the atmospheric energy thresholds and are further used to calculate $q(h)$ through integration over the cosmic ray spectrum.

The three-interval approximation thus improves the description of energy loss processes for low, medium, and high energies, ensuring a more accurate estimation of the ion production rate in the middle and upper atmosphere. The resulting formulation is implemented in the CORIMIA model and serves as the basis for subsequent calculations of ionization profiles under quiet and disturbed heliospheric conditions.

Three-Interval Approximation for Cosmic Ray Protons ($Z = 1$)

The ionization rate in the atmosphere depends strongly on the spectrum of primary cosmic ray particles and the energy-dependent ionization losses during their propagation. For charged particles with $Z = 1$ (protons), the energy loss function dE/dh is approximated by three characteristic intervals according to Equation (4). This approach extends the previous two-interval model by including a high-energy domain that improves the description above 5 GeV, where relativistic effects and deep penetration become significant.

Atmospheric Cut-off Energies and Depth Relations

The atmospheric cut-off represents the depth (or equivalent mass thickness) at which a cosmic ray particle of given kinetic energy E_k is completely absorbed or loses its ability to ionize the atmosphere effectively. For the three-interval approximation, the total penetration depth is expressed as the sum of contributions from each interval:

$$(8) \quad \bar{h} = \bar{h}_1 + \bar{h}_2 + \bar{h}_3 = \left(\frac{1}{242}\right) \int_{0.15}^{0.6} \left(\frac{dE}{E^{0.75}}\right) + \left(\frac{1}{2}\right) \int_{0.6}^{5.0} dE + \left(\frac{1}{0.7}\right) \int_{5.0}^{E_{a3}(h)} \left(\frac{dE}{E^{0.123}}\right).$$

Evaluating these integrals gives:

$$(9) \quad \bar{h} = \left(\frac{1}{423.75}\right) \left(600^{\frac{7}{4}} - 0.15^{\frac{7}{4}}\right) + \left(\frac{1}{2}\right) (5.0 - 0.6) + \left(\frac{1}{0.61}\right) (E_{a3}^{0.877} - 5.0^{0.877}).$$

From this relation, the atmospheric cut-off energy in the third interval is derived as:

$$(10) \quad E_{a3}(h) = (297.9 + 0.61\bar{h})^{1.14}.$$

This expression defines the lower energy boundary for protons penetrating into Interval 3 ($E \geq 5$ GeV).

Transition Between Intervals

When the particle energy passes through the boundaries at 600 MeV or 5 GeV, separate relations describe the behavior in the overlapping regions between intervals:

$$(11) \quad \bar{h} = \bar{h}_2 + \bar{h}_3 = \left(\frac{1}{2}\right) \int_{E_{32}(h)}^{5.0} dE + \left(\frac{1}{0.7}\right) \int_{5.0}^{E_k} \left(\frac{dE}{E^{0.123}}\right),$$

which yields the energy for the 5 GeV transition as:

$$(12) \quad E_{32}(h) = 5.0 - 2\bar{h} + 3.28(E_k^{0.877} - 5.0^{0.877}).$$

Similarly, when the particle crosses both interval boundaries (600 MeV and 5 GeV), the total depth is:

$$(13) \quad \bar{h} = \bar{h}_1 + \bar{h}_2 + \bar{h}_3 = \left(\frac{1}{242}\right) \int_{E_{32}(h)}^{0.6} \left(\frac{dE}{E^{0.75}}\right) + \left(\frac{1}{2}\right) \int_{0.6}^{5.0} dE + \left(\frac{1}{0.7}\right) \int_{5.0}^{E_k} \left(\frac{dE}{E^{0.123}}\right).$$

and the corresponding energy boundary becomes:

$$(14) \quad E_{31}(h) = \left(600^{\frac{7}{4}} - 423.75\bar{h} + 211.88(5.0 - 0.6) + 694.67(E_k^{0.877} - 5.0^{0.877})\right)^{\frac{4}{7}}.$$

Special Cases for Boundary Transitions

For the 600 MeV boundary, when the incoming particle energy lies in Interval 2:

$$(15) \quad \bar{h} = \left(\frac{1}{2}\right) \int_{0.6}^{E_{600,2}} dE = \left(\frac{1}{2}\right) (E_{600,2} - 0.6) \rightarrow E_{600,2} = 0.6 + 2\bar{h}.$$

When the energy belongs to Interval 3, the relation becomes:

$$(16) \quad \bar{h} = \left(\frac{1}{2}\right) (5.0 - 0.6) + \left(\frac{1}{0.61}\right) (E_{600,3}^{0.877} - 5.0^{0.877}),$$

and hence:

$$(17) \quad E_{600,3} = (0.61\bar{h} + 5.0^{0.877} - 0.3(5.0 - 0.6))^{1.14}.$$

Finally, for the 5 GeV boundary, when the particle enters from infinity:

$$(18) \quad \tilde{h} = \left(\frac{1}{0.7}\right) \int_{5.0}^{E_{5000,3}} \left(\frac{dE}{E^{0.123}}\right) = \left(\frac{1}{0.61}\right) (E^{0.877}_{5000,3} - 5.0^{0.877}).$$

and the corresponding relation is:

$$(19) \quad E_{5000,3} = (5.0^{0.877} + 0.61\tilde{h})^{1.14}.$$

Discussion

Figure 2 illustrates the three-interval approximation of the ionization energy loss function $(1/\rho)(dE/dh)$ for cosmic ray protons ($Z = 1$) in the CORIMIA model. Each colored region corresponds to a distinct energy domain:

- the low-energy interval (0.15–0.6 GeV), where the ionization losses follow a steep inverse power law,
- the intermediate interval (0.6–5 GeV), characterized by nearly constant specific losses, and
- the high-energy interval (> 5 GeV), where relativistic corrections modify the loss function toward a slow rise with energy.

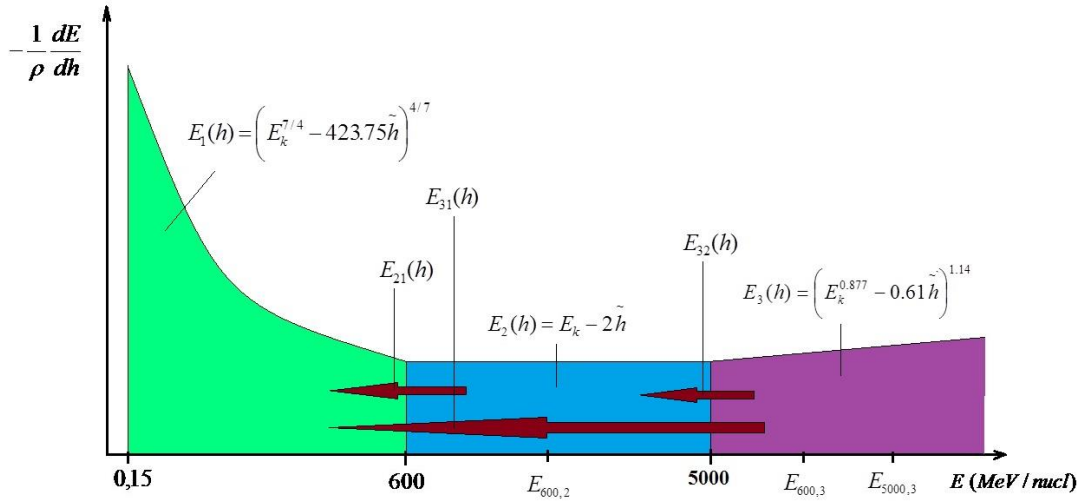


Fig. 2. Three-interval approximation of the ionization energy loss function for cosmic ray protons ($Z = 1$). The curves $E_1(h)$, $E_2(h)$, and $E_3(h)$ correspond to the laws of energy losses for the individual intervals, while $E_{21}(h)$, $E_{32}(h)$, and $E_{31}(h)$ represent the transition cases where the kinetic energy of the ionizing particles crosses one or two energy boundaries

The analytical relations $E_1(h)$, $E_2(h)$, and $E_3(h)$ describe the evolution of particle energy with atmospheric depth for each interval. The additional functions $E_{21}(h)$, $E_{32}(h)$, and $E_{31}(h)$ define the transitional cases when the penetrating proton crosses one or two boundaries between adjacent intervals. These relations ensure continuity of the energy–depth dependence and prevent artificial discontinuities in the total ionization rate $q(h)$, which was a limitation of earlier two-interval models.

Physically, the three-interval approach captures the different regimes of proton–air interactions.

At low energies, the stopping power is dominated by ionization and excitation processes following the Bethe–Bloch law; in the mid-energy range, the specific loss per unit path becomes nearly constant; and at relativistic energies, energy losses increase again due to radiative and density-effect corrections.

This behavior determines the shape of the ionization profile in the atmosphere, particularly above the Pfozter maximum.

By integrating the loss function over the differential proton spectrum $D(E)$, the CORIMIA model provides altitude-dependent ionization rates that are consistent with empirical data and Monte Carlo simulations [3, 4]. The implementation of the third energy interval improves the accuracy of the calculated ionization rate in the upper stratosphere and mesosphere, where the contribution of high-energy protons becomes significant. This extension is especially relevant for space weather studies, as it allows the model to simulate both quiet-time galactic cosmic ray ionization and event-driven enhancements from solar energetic particles (SEPs).

Conclusions

The three-interval approximation developed within the CORIMIA ionization model represents a significant refinement in the analytical treatment of cosmic ray-induced ionization. Compared with the earlier two-interval formulation, the inclusion of a high-energy domain (above 5 GeV) provides:

- improved continuity of the ionization loss function across energy boundaries,
- more realistic representation of relativistic energy losses,
- and enhanced accuracy of the ion production rate in the upper atmosphere.

The analytical expressions derived for $E_i(h)$ and $E_{ij}(h)$ yield compact formulas for the atmospheric cut-off energies and penetration depths. These relations can be used in both forward modeling and inverse reconstruction of particle energy spectra from observed ionization profiles. The CORIMIA model, supplemented by the present three-interval approximation, therefore offers a robust framework for studying ionization processes across the full range of cosmic ray proton energies, linking theoretical physics with practical space climate applications.

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